Math 102

Krishanu Sankar

September 11, 2018

Announcements

- OSH 0 due tomorrow, OSH 1 due Friday.
  - Consider a scanning app - Fast Scanner or TinyScanner.
  - OSHs may be submitted as a group of up to 4 students.
- Section canvas page is up - Office hours survey.
- MLC opens Friday Sept 14
  - Specific questions and finding study buddies
  - For more open-ended questions about OSH, come to OH or post to Piazza.

Last time

- Power Functions, **asymptotic behavior**
- A **polynomial** is a sum of any number of power functions

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

- Graphing simple polynomials $f(x) = Ax^m + Bx^n$, where $m > n$, and A, B are constants.
  - When x is very large, $Ax^m + Bx^n \approx Ax^m$.
  - When x is near 0, $Ax^m + Bx^n \approx Bx^n$.
  - Sketch in between.

Asymptotic Thinking

- Question: Which is the best approximation?

$$1,000,000 + 5,000 - 3 \approx$$

A) 1,000,000 B) -3 C) other

- Question: Which is the best approximation?

$$.000001 + .005 - 3 \approx$$

A) .000001 B) -3 C) other

Asymptotic Thinking

- Question: Which is the best approximation?

$$1,000,000 + 5,000 - 3 \approx$$

A) 1,000,000 B) -3 C) other

- Question: Which is the best approximation?

$$.000001 + .005 - 3 \approx$$

A) .000001 B) -3 C) other

Asymptotic Thinking

Question: Which is the best approximation?

$$3,000 \cdot 20 \approx$$

A) 3,000 B) 20 C) other

Asymptotic Thinking

Question: Which is the best approximation?

$$3,000 \cdot 20 \approx$$

A) 3,000 B) 20 C) other

$3,000 \cdot 20 = 60,000$. Our approximation method of bigger term wins doesn't work for multiplication.

Asymptotic Thinking

Question: How would you calculate a simple approximation?

$$\frac{7,129}{73} \approx$$

Question: How would you calculate a simple approximation?

$$\begin{aligned}\frac{7,129}{73} &= \frac{7,000+100+20+9}{70+3} \\ &\approx \frac{7,000}{70} \\ &= \boxed{100}\end{aligned}$$

Rational Functions

- A **rational function** is a quotient of two polynomials

$$f(x) = \frac{a_n x^n + \ldots + a_1 x + a_0}{b_m x^m + \ldots + b_1 x + b_0}$$

- Example on board: sketch

$$f(x) = \frac{3x}{4 + x}$$

Try it out!

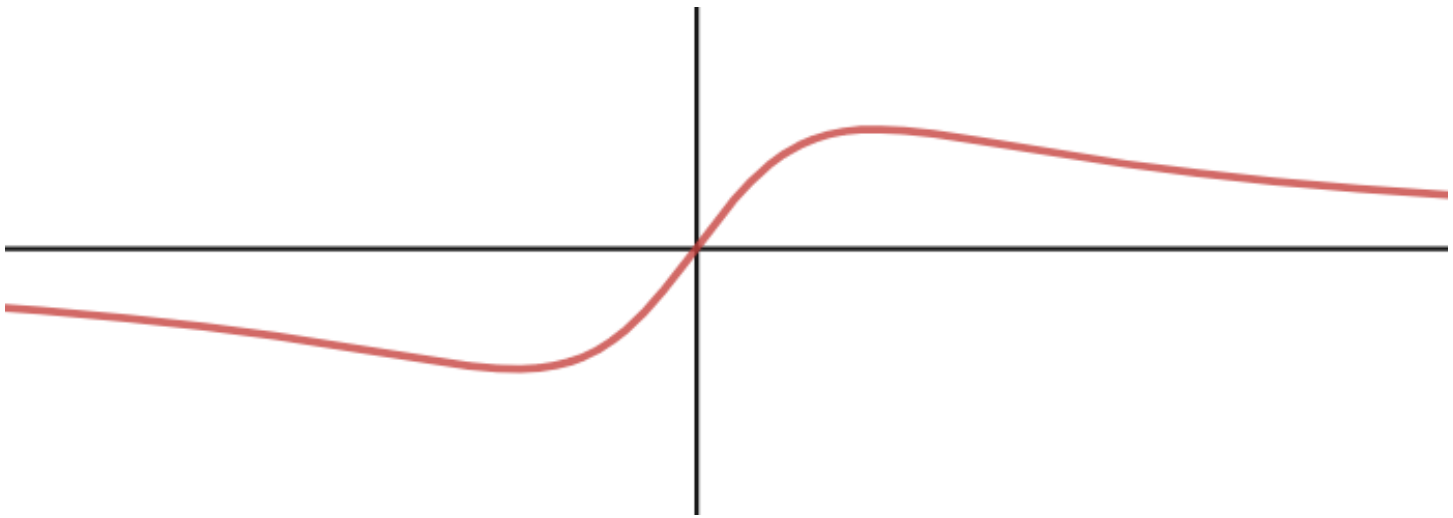
- Exercise: Sketch the graph of $f(x) = \frac{4x}{3+5x^2}$.
- Exercise: Give approximations for

$$f(1002) \approx$$

$$f(-.03) \approx$$

$$f(x) = \frac{4x}{3+5x^2}$$

- When x is large, $f(x) \approx \frac{4x}{5x^2} = \frac{4}{5x}$.
- When x is small, $f(x) \approx \frac{4x}{3}$.



$$f(x) = \frac{4x}{3+5x^2}$$

$$f(1002) = \frac{4(1002)}{3 + 5(1002)^2} \approx \frac{4(1002)}{5(1002)^2}$$

$$\approx \frac{4(1000)}{5(1000)^2} = \frac{4}{5000} = .0008$$

$$f(x) = \frac{4x}{3+5x^2}$$

$$f(1002) = \frac{4(1002)}{3+5(1002)^2} \approx \frac{4(1002)}{5(1002)^2}$$

$$\approx \frac{4(1000)}{5(1000)^2} = \frac{4}{5000} = .0008$$

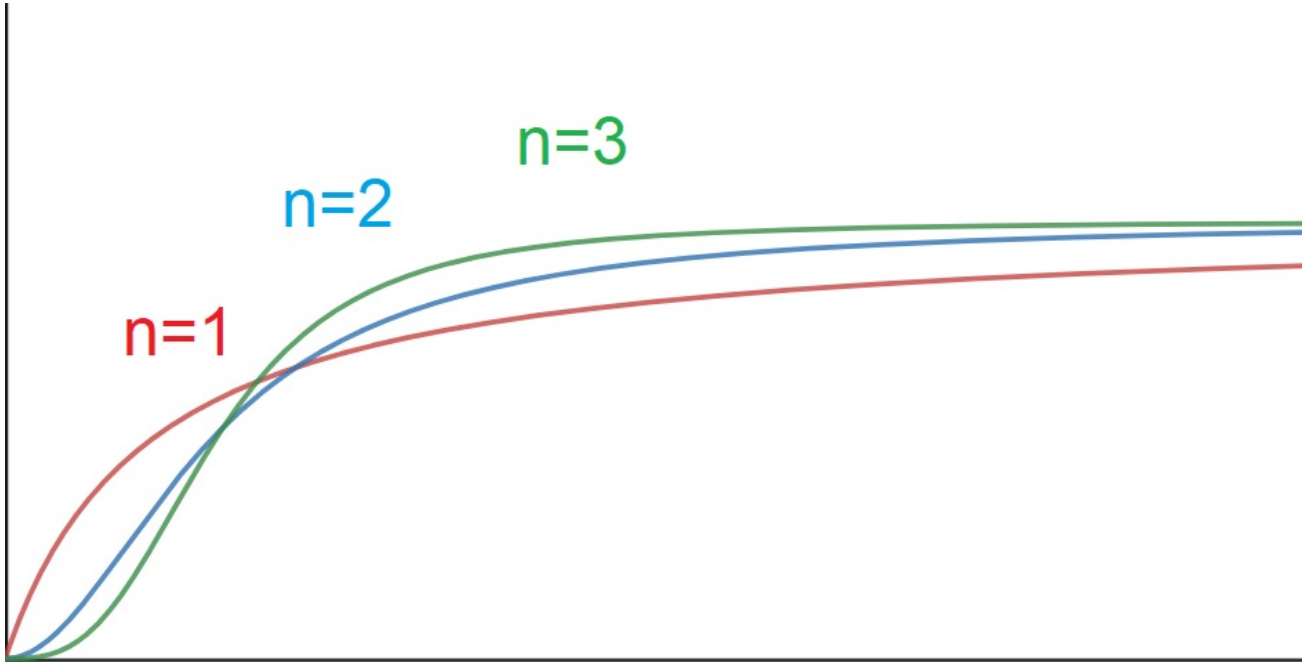
$$f(.03) = \frac{4(.03)}{3+5(.03)^2} \approx \frac{4(.03)}{3} = .04$$

Hill Functions (Archibald Hill, 2010)

- A **Hill function** is a function of the form

$$f(x) = \frac{Ax^n}{a^n + x^n}$$

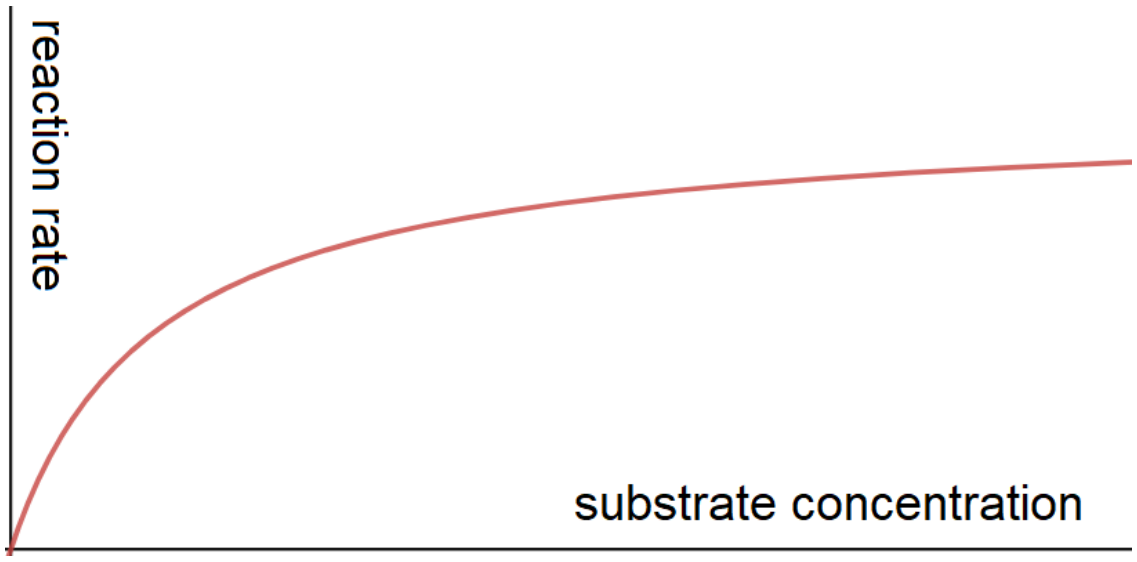
for some constants A, a, n.



Enzyme Kinetics, single substrate

$$E + S \rightleftharpoons ES \rightleftharpoons EP \rightleftharpoons E + P$$

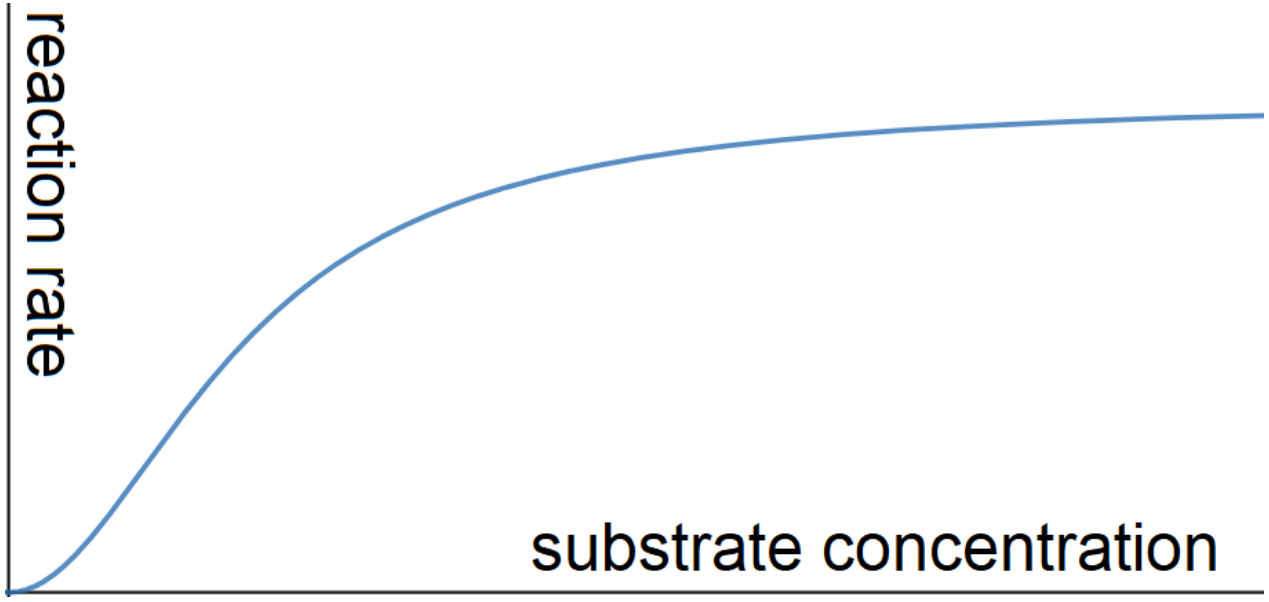
- Low substrate concentration - reaction rate is limited by substrate.
- High substrate concentration - reaction rate is limited by enzyme.



Enzyme Kinetics, hypothetical double substrate

$$E + 2S \rightleftharpoons ES + S \rightleftharpoons ESS \rightleftharpoons EPP \rightleftharpoons EP + P \rightleftharpoons E + 2P$$

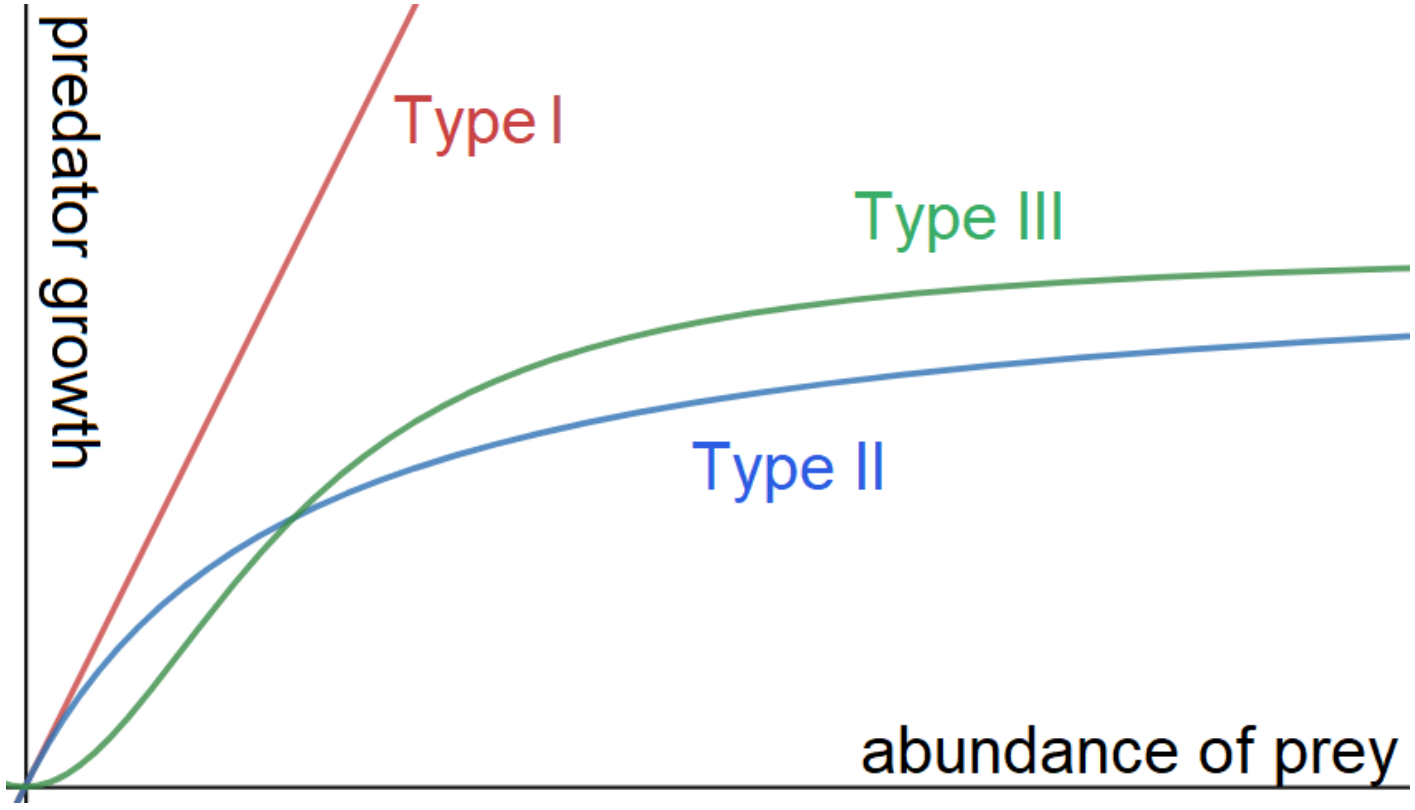
Two copies of substrate required for reaction - quadratic dependence on substrate concentration.



Hill Functions

https://www.desmos.com/calculator/dttwc7svsx

Holling Predator Response



Type I: $P(x) = Ax$

Type II: $P(x) = \frac{Ax}{a+x}$

Type III: $P(x) = \frac{Ax^n}{a^n+x^n}$, $n \geq 2$

Recap and Reminders

- Asymptotic approximation
  - Works for sums and differences.
  - Doesn't work for products and quotients!
- Sketching rational functions - approximate the numerator and denominator.
- Hill functions
  - n, A, a
  - Enzyme kinetics
  - Predator response

- Reminder - Office hours survey on canvas page.